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VIBRATIONS OF A FRAMELESS FILM MEMBRANE STABILIZED BY THE AMPÈRE FORCE IN ZERO GRAVITY

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Abstract: The dynamics of a circular film membrane with attached current-carrying conductors in zero gravity is studied. Equations are derived which describe the vibrations of the membrane stabilized by the Ampère force. The spectrum of natural vibrations and their corresponding strains are calculated. Constrained vibrations of the membrane are studied. The effect of the geomagnetic field on the stability of the membrane and the damping of its vibration is investigated for unsteady modes of application of mechanical forces in zero gravity.

Keywords: film membrane current-carrying conductor, Ampère force, natural vibrations of membrane, damping of vibrations.

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INTRODUCTION

Frameless flexible film structures are currently used in space technology [1]. The most important of these structures are solar-cell arrays which can be installed on small spacecraft or large orbital power stations and space vehicles. Use of thin films coated with amorphous silicon increases the power density of solar-cell arrays by a factor of more than ten [2, 3]. It is suggested that the deployment of film structures in space can be provided by the inertial forces arising from the rotation of the spacecraft [4, 5]. It has also been proposed [6, 7] to use the strength of the interaction of the magnetic field currents occurring in thin conductors disposed on the surface of film structures.

In addition to the problem of deployment, in the design of large flexible space structures there is the problem of retaining their given shape during operation. One way to stabilize the given shape of film structures is to place current-carrying conductors in the form of concentric circles on their surface. During spatial reorientation, the film structure is acted upon not only by the steady field of the Ampère forces, but also by unsteady inertia forces, resulting in the occurrence of vibrations of the film structure. With a virtually complete absence of external damping forces, these vibrations can exist for a long time. In this study, we investigated vibrational processes of a film structure.

1. CONSTRAINED VIBRATIONS OF A THIN MEMBRANE STABILIZED BY THE AMPÈRE FORCE

We consider the constrained vibrations of a flat circular membrane of radius R and thick h, whose bulk density will be denoted by ρ. Radial stress in the membrane is produced by the Ampère force resulting from the interaction of the concentric current-carrying conductors located on its surface. The transverse displacement of a membrane surface element w in polar coordinates (r, θ) satisfies the equation of transverse vibrations [8]

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$$\rho \frac{\partial^2 w}{\partial t^2} - \frac{1}{r}\frac{\partial}{\partial r}\Big(r\sigma_{rr}\frac{\partial w}{\partial r}\Big) - \frac{1}{r^2}\frac{\partial}{\partial \theta}\Big(\sigma_{\theta\theta}\frac{\partial w}{\partial \theta}\Big) = F(r,\theta,t), \tag{1}$$

where σ_{rr} and $\sigma_{\theta\theta}$ are the radial and tangential components of the stress tensor of the membrane, and F is the bulk density of the force directed normal to the plane of the membrane.

On the surface of the membrane are thin conductors with constant current strength I_i, having the form of concentric circles of radius r_i. The subscript i takes values $i = 1, 2, \ldots, N$. The radii of the circles increase with increasing circle number. The greatest radius of the conductor coincides with the radius of the membrane: $r_N = R$. According to the Ampère law, the magnetic field induction $\boldsymbol{B}$ of the current-carrying conductors leads to their interaction [9], which determines the stress in the membrane. The strain and stress in the membrane were calculated in [7]. The stress tensor components are given by formulas

$$\sigma_{rr} = \frac{E}{1-\sigma^2}\Big((1+\sigma)k_i - (1-\sigma)\frac{p_i}{r^2}\Big), \qquad \sigma_{\theta\theta} = \frac{E}{1-\sigma^2}\Big((1+\sigma)k_i + (1-\sigma)\frac{p_i}{r^2}\Big),$$

$$r_i < r < r_{i+1},$$

where E is Young's modulus, σ is Poisson's ratio, and the coefficients k_i and p_i are given by

$$k_n = \frac{g_N}{1+\sigma} + \frac{1-\sigma}{2(1+\sigma)R^2}\sum_{i=1}^{N-1} g_i r_i^2 + \frac{1}{2}\sum_{i=n+1}^{N-1} g_i,$$

$$p_0 = 0, \qquad p_n = \frac{1}{2}\sum_{i=1}^{n} g_i r_i^2, \quad n = 0, \ldots, N-1,$$

$$g_i = \frac{\mu_0(1-\sigma^2)}{4\pi E h} I_i\Big[\frac{I_i}{r_i}\Big(\ln\frac{8r_i}{a} - \frac{3}{4}\Big) + 2\sum_{j\neq i} I_j\Big(\frac{1}{r_j + r_i}K(\lambda_{ij}) + \frac{1}{r_j - r_i}E(\lambda_{ij})\Big)\Big],$$

$$\lambda_{ij} = \frac{4r_i r_j}{(r_i + r_j)^2},$$

$K(x)$ and $E(x)$ are the Legendre complete elliptic integrals [10] and μ_0 is the magnetic permeability of vacuum, a is the cross-section radius of the conductors located on the membrane. It should be noted that the continuity of the dependence of the stress on the radius is broken for values of the radius $r = r_i$.

Equation (1) with variable coefficients is solved with the boundary conditions at the center of the membrane $r = 0$ and on the circle $r = R$ bounding the membrane:

$$\frac{\partial w}{\partial r}\Big|_{r=0} = 0, \qquad \Big[\frac{\partial^2 w}{\partial r^2} + \frac{\sigma}{r}\Big(\frac{\partial w}{\partial r} + \frac{1}{r}\frac{\partial^2 w}{\partial \theta^2}\Big)\Big]\Big|_{r=R} = 0. \tag{2}$$

For $r = r_i$, the displacement w is continuous and the derivative $\partial w/\partial r$ has a discontinuity, whose value is determined by integrating Eq. (1) over r in the neighborhood of the radius $r = r_i$. We assume that at $t < 0$, the force $F = 0$ and there are no vibrations of the membrane. The initial conditions for the transverse displacement and the conditions on the circle $r = r_i$ are of the form

$$\begin{gathered} w\Big|_{t=0} = 0, \qquad \frac{\partial w}{\partial t}\Big|_{t=0} = 0, \\ \Big[\sigma_{rr}\frac{\partial w}{\partial r}\Big]\Big|_{r=r_i} = 0, \qquad [w]\Big|_{r=r_i} = 0, \end{gathered} \tag{3}$$

where $[\,\cdot\,]$ is the discontinuity of the corresponding function for $r = r_i$.

Suppose that the density of the external force acting on the membrane is given by the series

$$F(r,\theta,t) = \sum_{n=0}^{\infty} f_n(r,t)\cos(n\theta).$$

Then, the solution of Eq. (1) can be represented as

$$w(r,\theta,t) = \sum_{n=0}^{\infty} u_n(r,t)\cos(n\theta). \tag{4}$$

For the dimensionless functions $U_n = u_n/R$, from (1) we obtain the equation

$$\frac{\partial^2 U_n}{\partial \tau^2} + \hat{L}_n(U_n) = \tilde{f}_n(\xi,\tau), \qquad \hat{L}_n(U_n) = -\frac{1}{\xi}\frac{\partial}{\partial \xi}\Big(\xi s_1(\xi)\frac{\partial U_n}{\partial \xi}\Big) + \frac{n^2}{\xi^2}s_2(\xi)U_n,$$
$$s_1(\xi) = \sigma_{rr}(\xi)\frac{1-\sigma^2}{E}, \qquad s_2(\xi) = \sigma_{\theta\theta}(\xi)\frac{1-\sigma^2}{E}, \qquad \tilde{f}_n = f_n\frac{R(1-\sigma^2)}{E}, \tag{5}$$

where $\xi = r/R$, $\tau = t/t_0$, and $t_0 = R\sqrt{\rho(1-\sigma^2)/E}$ are dimensionless variables. The solution of Eqs. (5) is sought in the form of expansions of the unknowns U_n in the systems of eigenfunctions of the operators $\hat{L}_n$:

$$U_n(\xi,\tau) = \sum_{m=0}^{\infty} a_{mn}(\tau)u_{mn}(\xi).$$

The orthogonal functions $u_{mn}(\xi)$ satisfy the equations

$$\frac{d}{d\xi}\Big(\xi s_1(\xi)\frac{du_{mn}}{d\xi}\Big) + \Big(\lambda_{mn}^2\xi - \frac{n^2 s_2(\xi)}{\xi}\Big)u_{mn} = 0 \tag{6}$$

and the boundary conditions (2) at the center of the membrane and on its boundary circle. The expansion coefficients $a_{mn}(\tau)$ satisfy the equations and initial conditions

$$\frac{d^2 a_{mn}}{d\tau^2} + \lambda_{mn}^2 a_{mn} = b_{mn}(\tau),$$

$$b_{mn}(\tau) = \int_0^1 \xi \tilde{f}_n(\xi,\tau)u_{mn}(\xi)\,d\xi, \quad a_{mn}(0) = \frac{da_{mn}(\tau)}{d\tau}\Big|_{\tau=0} = 0.$$

The eigenvalues λ_{mn} define the eigenfrequencies and vibration periods of the membrane: $\omega_{mn} = \lambda_{mn}/t_0$ and $T_{mn} = 2\pi t_0/\lambda_{mn}$.

Problem (6) for the eigenfunctions and eigenvalues was solved numerically by the shooting method [11]. Since the dimensionless stress tensor components $s_{1,2}(\xi)$ in (6) have a discontinuity for $\xi = \xi_i = r_i/R$, Eq. (6) was solved sequentially in the intervals $\xi_i < \xi < \xi_{i+1}$ starting from the center of the membrane $\xi_0 = 0$ and using conditions (3) on the boundaries of these intervals.

Figure 1 shows the calculated curves of the eigenfunctions of transverse vibrations of the membrane versus radius for $n = 1$, $m = 0$, 1, 2, and 3, and the corresponding shapes of the membrane surface $u_{m1}(\xi)\cos\theta$. The harmonic numbers with $m = 0$, 1, 2, and 3 correspond to vibration periods $T = 80.0$, 40.0, 27.8, and 20.9 s. The calculation was performed for a membrane with three concentric ring conductors. The current values were chosen so as to provide zero total magnetic moment currents in order to minimize the effect of the external magnetic field on the membrane. The calculation results show that the period of the fundamental vibration harmonic is 80 s. Its value depends on the current values in the conductors.

2. VIBRATIONAL EXCITATION OF THE MEMBRANE STABILIZED BY THE AMPÈRE POWER

We investigate the vibrational excitation of the circular membrane relative to the fixed axis. The axis of rotation is located in the plane of the membrane and passes through its center. Vibrations result from the rotation of the membrane with a variable angular velocity $\Omega(t)$ with respect to the axis of rotation. The plane of the membrane coincides with the plane (x, y) of the Cartesian coordinate system whose origin coincides with the center of the membrane. The axis of rotation is directed along the y axis. The angle θ is measured from the x axis. The radius vector $\boldsymbol{r}$ lies in the plane (x, y). The rotation gives rise to an inertial force $\Delta\boldsymbol{f}$, which acts on each membrane element of mass $\Delta m = \rho h \Delta S$ with a surface area of ΔS. We find the transverse component of the bulk density of the inertial force $F = \Delta f_\perp/(h\Delta S)$ responsible for the transverse displacements w of the membrane. The inertial force acting on a small element of the membrane is given by the expression (see, e.g., [12])

$$\Delta\boldsymbol{f} = -\Delta m[(\dot{\boldsymbol{\Omega}} \times \boldsymbol{r}) + 2(\boldsymbol{\Omega} \times \dot{\boldsymbol{r}}) + (\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{r}))].$$

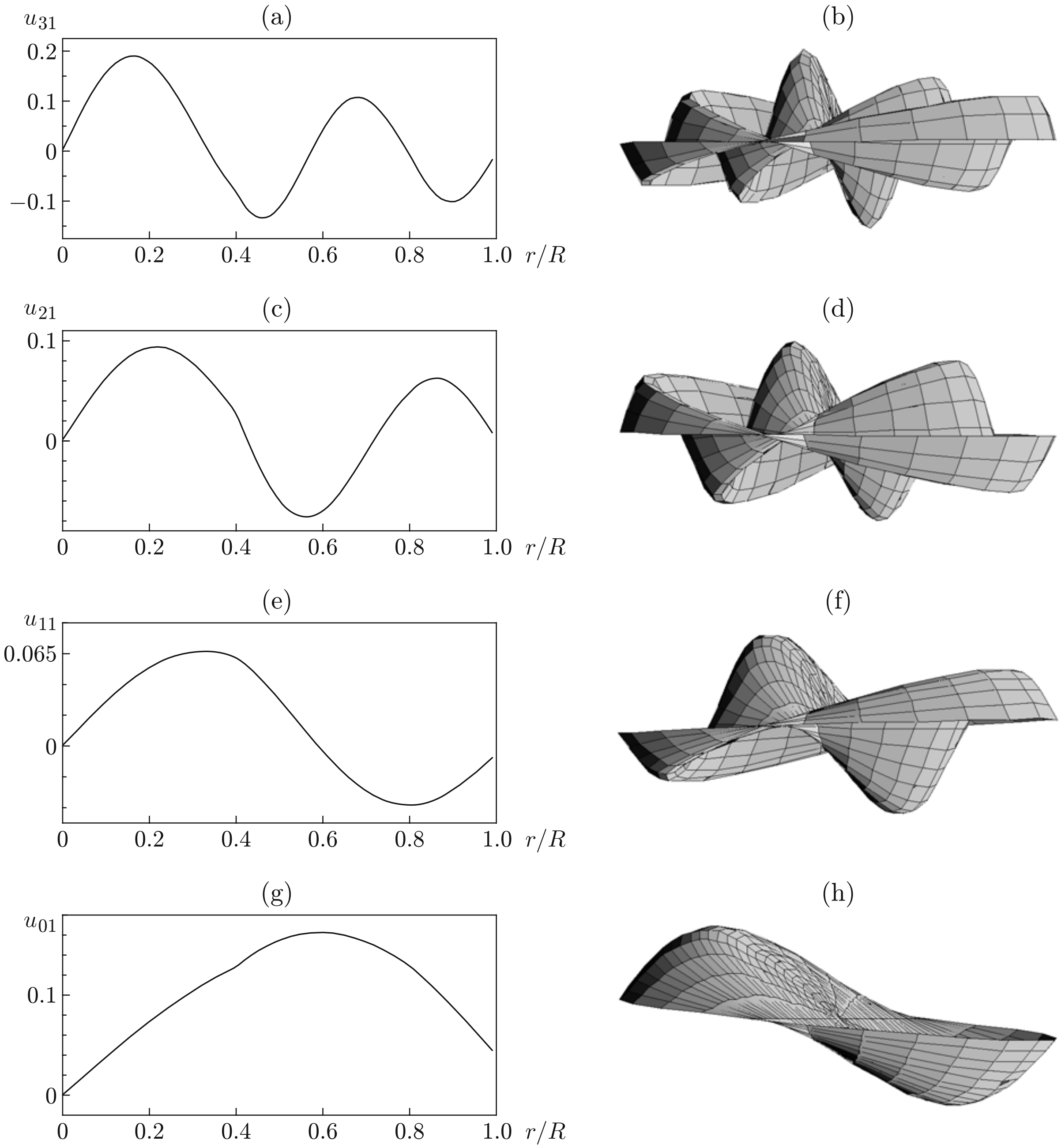

Fig. 1. Calculated eigenfunctions of the transverse vibrations of the membrane for four harmonics ($E = 3 \cdot 10^9$ N/m^2, $\sigma = 0.3$, $h = 5 \cdot 10^{-6}$ m, $\rho = 2 \cdot 10^3$ kg/m^3, $a = 10^{-4}$ m, $R = 1$ m, $r_1 = 0.4$ m, $r_2 = 0.8$ m, $r_3 = 1.0$ m, $I_1 = 1.0$ A, $I_2 = 2.0$ A, and $I_3 = -1.44$ A): (a, c, e, g) displacement of the membrane surface element versus radius; (b, d, f, h) shape of the surface of the membrane; (a, b) $m = 3$ and $\lambda = 96$; (c, d) $m = 2$ and $\lambda = 72$; (e, f) $m = 1$ and $\lambda = 48$; (g, h) $m = 0$ and $\lambda = 25$.

In this expression, the second and third terms are the Coriolis force and the centrifugal force acting in the plane of the membrane. The first term describes the inertia of rotation, directed perpendicular to the plane of the membrane. Using the normal component of this term, we can obtain the transverse component of the force acting on unit volume that results from rotation of the membrane around the y axis with a constant angular velocity $\Omega(t)$:

$$F = \rho r \frac{d\Omega}{dt} \cos\theta. \tag{7}$$

We analyze the transverse vibrations of the membrane using Eq. (1) and boundary conditions (2) and (3). If the rotation of the plane of the membrane around a fixed axis occurs with a variable angular velocity $\Omega(t)$, it is acted upon by the inertial force (7). Substitution of (7) into (1) yields the following hyperbolic equation and the initial and boundary conditions for the function $u(r,t)$:

$$\begin{gathered} \rho \frac{\partial^2 u}{\partial t^2} - \frac{1}{r}\frac{\partial}{\partial r}\Big(r\sigma_{rr}\frac{\partial u}{\partial r}\Big) + \frac{\sigma_{\theta\theta}}{r^2} u = f(t)r, \qquad f(t) = \rho\frac{d\Omega(t)}{dt}, \\ u\Big|_{t=0} = \frac{\partial u}{\partial t}\Big|_{t=0} = 0, \qquad u\Big|_{r=0} = 0, \qquad \Big[\frac{\partial^2 u}{\partial r^2} + \frac{\sigma}{R}\Big(\frac{\partial u}{\partial r} - \frac{u}{R}\Big)\Big]\Big|_{r=R} = 0. \end{gathered} \tag{8}$$

System (8) is a special case of system (2), (3), (5), for which an exact solution was obtained in Section 1 by expanding in eigenfunctions. However, for a linear dependence of the external force on the radius, this method is of little use because of the very slow convergence of the corresponding series. At the same time, the results of direct numerical simulations show that the function $u(r,t)$ can be represented with high accuracy as a linear function. Therefore, we assume that

$$u(r,t) = g(t)r. \tag{9}$$

In this case, the boundary conditions at $r = 0$ and $r = R$ are automatically satisfied. Next, we need to determine $g(t)$. Substitution of (9) into Eq. (8) is not possible since this function is not a solution. To determine $g(t)$, we substitute (9) into the energy integral (see, e.g., [13]), which is a consequence of Eq. (8). To do this, we multiply Eq. (8) by $r\,\partial u/\partial t$ and integrate the result over r. After integration by parts taking into account the boundary conditions, we obtain

$$\begin{gathered} \frac{d}{dt}\int_0^R W r\,dr = \int_0^R [rf(t)]\frac{\partial u}{\partial t} r\,dr + R\sigma_{rr}\frac{\partial u}{\partial t}\frac{\partial u}{\partial r}\Big|_{r=R}, \\ W = \frac{1}{2}\Big[\rho\Big(\frac{\partial u}{\partial t}\Big)^2 + \sigma_{rr}\Big(\frac{\partial u}{\partial r}\Big)^2 + \frac{1}{r^2}\sigma_{\theta\theta}u^2\Big]. \end{gathered} \tag{10}$$

Equality (10) defines the balance of the mechanical energy density W of the vibrating membrane. The first term on the right side represents the work of the external force per unit time, and the second term is defined by the boundary conditions. Substituting the approximation solutions (9) into Eq. (10) and integrating the resulting equation, we obtain

$$\frac{d^2 g}{dt^2} + \omega_0^2 g = \frac{d\Omega(t)}{dt}, \qquad \omega_0 = \frac{4}{\rho R^4}\Big(\int_0^R (\sigma_{rr} + \sigma_{\theta\theta})r\,dr - R^2\sigma_{rr}(R)\Big). \tag{11}$$

Equation (11) coincides with the equation of vibrations of a harmonic oscillator with eigenfrequency ω_0 under the action of the external force $d\Omega/dt$. The quantity $g(t)$ is the maximum angle of deflection of the membrane at the time t. Let us calculate the constrained vibrations for the time dependence of the angular velocity

$$\Omega(t) = 2\Omega_0\Big[\exp\Big(-\frac{t}{\tau}\Big) - \exp\Big(-\frac{2t}{\tau}\Big)\Big].$$

In this case, the total angle of rotation of the axis is equal to $\Phi = \Omega_0\tau$, and the characteristic time of rotation is $t \approx 2\tau$. The natural frequency is $\omega_0 = 0.18\ \mathrm{s}^{-1}$, which corresponds to a period of vibrations $T = 2\pi/\omega = 34$ s. The results of calculation of the constrained vibrations of the membrane are shown in Fig. 2. The current values are chosen so as to provide zero total magnetic moment of the system in order to minimize the effect of the geomagnetic field. From Fig. 2 it follows that the maximum deflection angle of the membrane is not greater than 1°, and its steady-state value is somewhat smaller.

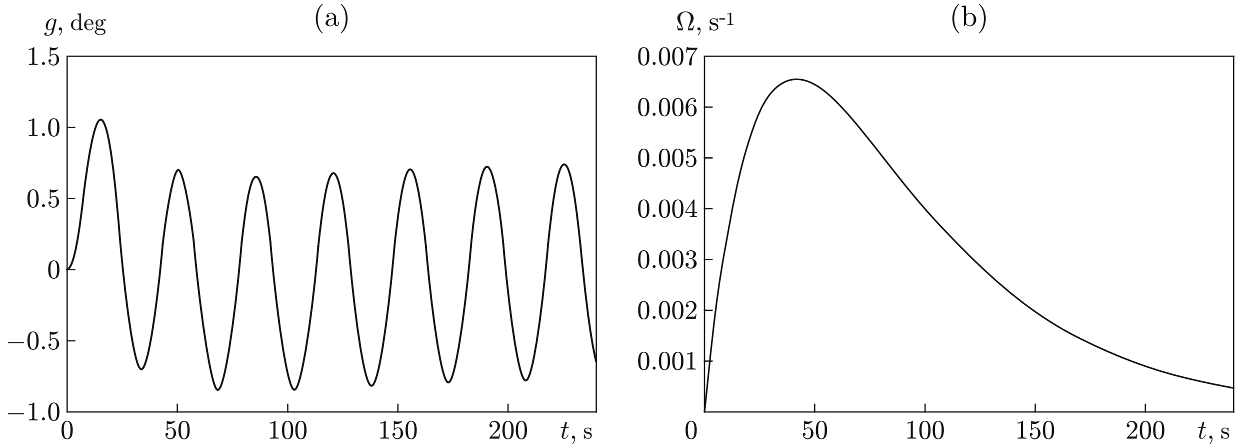

Fig. 2. Maximum angle of deflection of the membrane (a) and the angular velocity (b) versus time for $\Phi = \pi/2$, $\tau = 60$ s, $E = 3 \cdot 10^9$ N/m^2, $\sigma = 0.3$, $h = 5 \cdot 10^{-6}$ m, $\rho = 2 \cdot 10^3$ kg/m^3, $a = 10^{-4}$ m, $R = 1$ m, $r_1 = 0.4$ m, $r_2 = 0.8$ m, $r_3 = 1.0$ m, $I_1 = 4.0$ A, $I_2 = 8.0$ A, $I_3 = -5.76$ A, and $\omega_0 = 0.18$ s^{-1}.

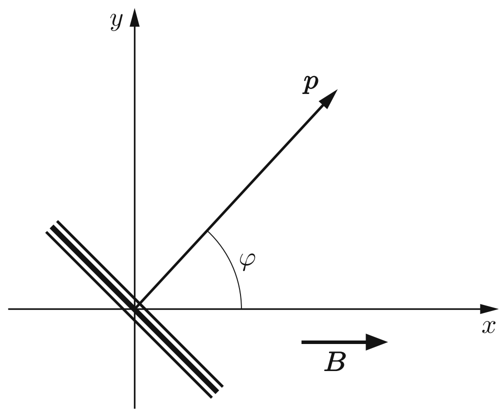

Fig. 3. Coordinate system in the problem of damping of vibrations of the membrane.

3. ELECTRODYNAMIC DAMPING OF VIBRATIONS OF A FLEXIBLE SOLAR-CELL ARRAY

A flexible power plant in the form of a film structure operating in space consists of solar cells located on a thin film and connected in a closed circuit. During flight, the spacecraft orients the plane of the solar-cell array toward the Sun. As a result, the normal vector to the plane of the solar-cell array is at an angle to the geomagnetic field. The closed circuit of the solar-cell array carries an electric current, which generates its magnetic moment. The magnetic moment vector of the solar cell is directed along the normal to its plane at an angle to the geomagnetic field. The state with the least energy corresponds to the orientation of the magnetic moment along the geomagnetic field. Hence, the magnetic moment tends to rotate the solar-cell array to a position where its plane is located along the normal to the geomagnetic field. The array is acted upon by a compensating moment to retain the orientation to the Sun. Perturbation of this moment, e.g., due to correction of the orbit, gives rise to mechanical vibrations of the array, which must be damped. The electrodynamic mechanism of this damping is considered below.

To evaluate the characteristics of the vibrations, we assume that the solar array is a circular thin membrane of radius r, along the edge of which is a closed circuit of solar power cells. Electric current I flows through the ring formed by these cells. An annular closed conductor of thickness h and width l is located on the opposite side of the nonconducting membrane along its edge. We introduce a Cartesian coordinate system x, y, z in which the x direction coincides with the direction of the geomagnetic field vector $\boldsymbol{B}$. The magnetic moment vector of the array $\boldsymbol{p}$ lies in the plane x, y at an angle φ to the x axis (Fig. 3). The vector of the moment of mechanical forces $\boldsymbol{M}$ that compensates the interaction of the magnetic moment with the geomagnetic field is directed along the z axis. The equation describing the rotational motion of the array about the z axis is written as

$$J\,\frac{d^2\varphi}{dt^2} = M - pB\sin\varphi, \tag{12}$$

where J is the moment of inertia of the array. We denote by M_1 a small perturbation of the moment with respect to its equilibrium value M_0. The perturbation of the moment leads to small changes in the angle φ_1 and the magnetic moment p_1. Substituting the expressions

$$M = M_0 + M_1, \qquad p = p_0 + p_1, \qquad \varphi = \varphi_0 + \varphi_1$$

into Eq. (12) and retaining terms of the first order of smallness with respect to the perturbations, we obtain

$$M_0 - p_0 B\sin\varphi_0 = 0, \qquad J\,\frac{d^2\varphi_1}{dt^2} = M_1 - (p_0 B\cos\varphi_0)\varphi_1 - B\sin\varphi_0 p_1. \tag{13}$$

The rotational vibrations of the membrane relative to the z axis result in a perturbation of the magnetic flux Φ and the excitation of the induction emf in the ring conductor. This electric current that arises in this case produces a perturbation of the magnetic moment:

$$p_1 = -\frac{S}{R_w}\,\frac{d\Phi}{dt} = -\frac{BS^2\sin\varphi_0}{R_w}\,\frac{d\varphi_1}{dt} \tag{14}$$

(S the area bounded by the ring and R_w is the resistance of the ring conductor). Substituting equality (14) into the second equation of (13), we obtain the equation of the vibrations of the membrane relative to the z axis:

$$\frac{d^2\varphi_1}{dt^2} + \frac{1}{\tau}\,\frac{d\varphi_1}{dt} + \Omega^2\varphi_1 = \frac{M_1}{J}, \qquad \tau = \frac{R_w J}{(BS\sin\varphi_0)^2}, \quad \Omega = \sqrt{\frac{p_0 B\cos\varphi_0}{J}}\,. \tag{15}$$

The solution of Eq. (15) has the form

$$\varphi_1(t) = \frac{i}{\omega_1 - \omega_2}\int\limits_{-\infty}^{t} dt'\,A(t')\{\exp\,[i\omega_1(t-t')] - \exp\,[i\omega_2(t-t')]\},$$
$$\omega_1 = -\frac{i}{2\tau} + \sqrt{\Omega^2 - \frac{1}{4\tau^2}}\,, \qquad \omega_2 = -\frac{i}{2\tau} - \sqrt{\Omega^2 - \frac{1}{4\tau^2}}\,, \tag{16}$$

where $A(t) = M_1(t)/J$.

We assume that the conductor located on the membrane surface has a ring shape with a radius R equal to the radius of the membrane, and that the radius of the cross section of the conductor is equal to a. If the resistivity of the material is equal to $1/\sigma_w$, where σ_w is its conductivity, the resistance of the conductor is $R_w = 2R/(\sigma_w a^2)$. The moment of inertia of the membrane relative to the axis coinciding with its diameter is equal to $J = mR^2/4$. We also assume that the mass of the conductor is greater than the mass of the membrane. If we denote by ρ_w the density of the conductor material, we obtain $m = 2\pi^2\rho_w R a^2$. Substituting these quantities into (15), we have

$$\tau = \frac{\rho_w}{\sigma_w(B\sin\varphi_0)^2}, \qquad \Omega = \sqrt{\frac{2IB\cos\varphi_0}{\pi\rho_w R a^2}}\,.$$

The relaxation time of the vibrations does not depend on the size of the structure.

We consider the membrane vibrations resulting from the action of the moment caused by, for example, the start of operation of vernier engines. If the operation time is less than the period of vibration, the dependence of the perturbation of the moment on time can be represented as a δ-function: $A(t) = A_0\delta(t)$. Substitution of this function into Eq. (16) yields

$$\varphi(t) = A_0\,\frac{2\tau}{\sqrt{(2\tau\Omega)^2 - 1}}\exp\Big(-\frac{t}{2\tau}\Big)\sin\Big(\sqrt{(2\tau\Omega)^2 - 1}\,\frac{t}{2\tau}\Big). \tag{17}$$

Figure 4a shows the dependence $G_1(t) = \varphi(t)/A_0$ obtained using formula (17). It is seen that the membrane performs damped vibrations. The regularities of vibrations of the membrane in the transition mode can be

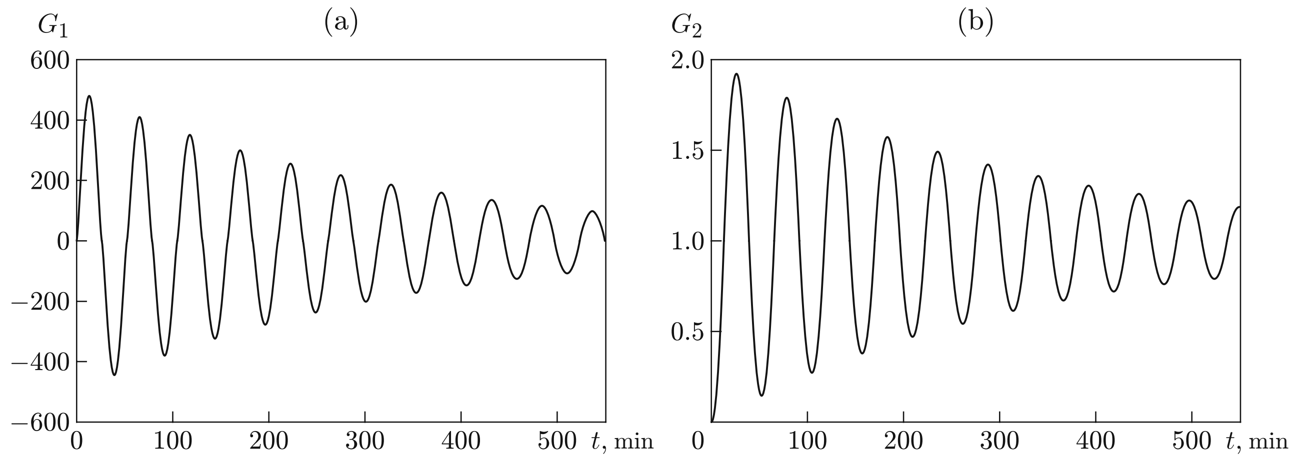

Fig. 4. Calculated dependence of the rotation angle of the membrane on time during its rotational vibrations ($1/\sigma_w = 1.7 \cdot 10^{-8}\ \Omega \cdot$m, $\rho_w = 8.9 \cdot 10^3$ kg/m^3, $B = 10^{-4}$ T, $\varphi_0 = \pi/4$, $R = 10$ m, $a = 5 \cdot 10^{-3}$ m, $\Omega = 2 \cdot 10^{-3}$ s^{-1}, $\tau = 10^4$ s): (a) $A(t) = A_0\delta(t)$ and $G_1(t) = \varphi(t)/A_0$; (b) $A(t) = A_0\eta(t)$ and $G_2(t) = \varphi(t)\Omega^2/A_0$.

determined from (16) by substituting the time dependence of the perturbation of the moment in the form of the Heaviside function $A(t) = A_0\eta(t)$. As a result, we obtain

$$\varphi(t) = \frac{A_0}{\Omega^2}\Big\{1 - \exp\Big(-\frac{t}{2\tau}\Big)\Big[\cos\Big(\sqrt{(2\Omega\tau)^2 - 1}\,\frac{t}{2\tau}\Big) + \frac{1}{\sqrt{(2\Omega\tau)^2 - 1}}\sin\Big(\sqrt{(2\Omega\tau)^2 - 1}\,\frac{t}{2\tau}\Big)\Big]\Big\}. \tag{18}$$

Figure 4b gives the function $G_2(t) = \varphi(t)\Omega^2/A_0$ obtained by formula (18). It is seen that the membrane performs damped vibrations about the equilibrium value of the angle of rotation $\varphi_0 + A_0/\Omega^2$. We consider the spectral characteristics of the membrane that performs rotational vibrations. Expanding the functions in (16) in the Fourier integral

$$\varphi(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\,\exp(-i\omega t)\varphi(\omega), \qquad A(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\,\exp(-i\omega t)A(\omega),$$

we obtain

$$G(\omega) = \frac{|\varphi(\omega)|}{|A(\omega)|} = \frac{1}{\sqrt{(\omega^2 - \Omega^2)^2 + (\omega/\tau)^2}}.$$

Figure 5 shows the dependence of the function G on frequency. The function has a maximum at a frequency $\omega = 2 \cdot 10^{-3}$ s^{-1}, which corresponds to a period of resonance $T = 2\pi/\omega = 50$ min.

CONCLUSIONS

The proposed method for calculating the vibrations of a frameless thin film membrane stabilized by the Ampère force in zero gravity can be used to determine the eigenfrequencies of vibrations of the membrane. This method allows one to choose the minimum value of the current in the conductors for which the period of the fundamental harmonic of transverse vibrations of the membrane will be larger than the period of its rotation. In this case, the shape of the membrane changes only slightly.

Analysis of the constrained vibration of the membrane shows that the use of the proposed method of stabilizing film structures in space by the Ampère force can reduces the parasitic vibrations resulting from the action of perturbing forces on the film. Therefore, the proposed method is preferred over the method of stabilizing

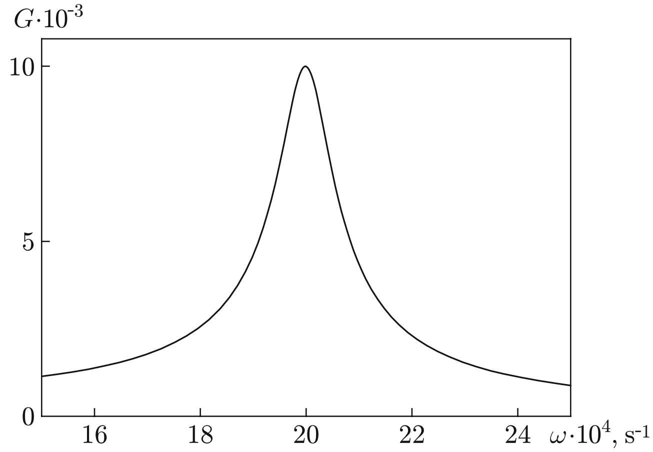

Fig. 5. Dependence of the function G on frequency ($1/\sigma_w = 1.7 \cdot 10^{-8}\ \Omega \cdot \mathrm{m}$, $\rho_w = 8.9 \cdot 10^3$ kg/m^3, $B = 10^{-4}$ T, $\varphi_0 = \pi/4$, $R = 10$ m, $a = 5 \cdot 10^{-3}$ m, $\Omega = 2 \cdot 10^{-3}$ s^{-1}, and $\tau = 10^4$ s).

by centrifugal forces. The vibration amplitude can be reduced by choosing such parameters of the structure that the frequency of the fundamental mode is higher the frequency of the perturbing force. The influence of the geomagnetic field on the moving film structure gives rise to resonance during vibrational rotation about the axis whose direction coincides with the direction of the vector of the momentum of mechanical forces that do not allow a change in the orientation of the film array toward the Sun. The period of resonant vibrations can be close in order of magnitude to the period of rotation of the film array relative to the Earth. These vibrations damp due to the excitation of an induction emf in the closed conducting circuit located on the surface of the film and due to heat release.

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