# ON THE ENVELOPE SOLITON – LIKE FINE STRUCTURE IN SOLAR RADIO EMISSION

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**Abstract.** Several quasi-periodic, milliseconds fine structures in the metric wave band occurring during the evolution of solar type IV bursts have been observed by Yunnan Radio Telescope, Trieste Radio Telescope and IZMIRAN dynamic spectrometer. The envelope of these quasi-period modulational fine structures have a soliton pattern, so it is called an envelope soliton-like fine structure. A modulational instability model of electromagnetic wave has been adopted here. It is found that the longitudinal modulational instability can occur only in the solar coronal region of low magnetic field and high temperature, as well as high density plasma, which will give rise to the envelope soliton-like fine structures in the solar metric and decimetric radio emission. The propagation effects of envelope soliton-like fine structure from corona to the observer on the Earth have been briefly considered.

#### 1. Introduction

Milliseconds fine structures are one of the recent interest in the observation and theoretical investigation of solar radio bursts. Most of these fine structures were considered to be related to the wave-wave and wave-particle interaction processes in the coronal plasma. For example, the fiber burst was generally explained by the coupling of Langmuir waves with whistler waves (Kuijpers, 1980); the radio spikes of decimetric waves was explained by a three-wave resonance interaction modulation, caused by their charactristic of coherent emission (Wang and Li, 1991; Wang *et al.*, 1997); the zebra pattern was proposed to be explained by whistler wave packets propagating obliquely to the magnetic field (Chernov, 1976; 1990).

On the other hand, some soliton models were used for fine structures as periodic fiber bursts in the type IV continuum radio bursts. Treumann and Bernold (1981) and Bernold and Treumann (1983), supposed that these periodic fiber bursts were related to the whistler soliton. However, in order to explain usual observed flux density of fiber-bursts  $\simeq 10^{-19}$  W m<sup>-2</sup> Hz and brightness temperatures  $-10^{14}$  K, it is necessary to combine the emission of about  $10^{14}$  solitons from the value about  $10^{8}$  km<sup>3</sup>. Treumann *et al.* (1990) had proposed the obliquely propagating Alfvén wave solitons to explain the solar intermediate drift bursts. These solitons may modulate the continuum emission. But taking into account conclusion of



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Galeev and Oraevsky (1963), a proposal about longitudinal propagation of high amplitude Alfvén waves is not real in most cases of astrophysical hypothesis due to the conversion of Alfvén waves into plasma oscillations of type fast and slow magnetosonic waves.

In fact, the wave energy density is able to grow the oscillating wave fields, it exert an average pressure and generate a ponderomotive force in plasma (Swenson, 1989). It might give rise to an envelope soliton structure by means of the modulation of low frequency wave. Possible candidates of such envelope soliton were observed by satellites (Gurnett *et al.*, 1981; Eriksson *et al.*, 1994; Wang *et al.*, 2002) in different space region. These envelope solitons were explained by modulational instability, which may be described by a nonlinear Schrödinger equation. (Chang, 1993; Guglielmi and Pokhotelov, 1994). Therefore, it is an interesting problem whether such modulated quasi-periodic fine structure whether can occur in the solar radio emission (Wang, Huang and Mao, 1997).

In this connection, we will try to investigate more real processes for solar corona when a strong emissivity of radio waves could form envelope soliton structures due to the modulation instability, responsible for some fine structures in the strong type IV radio continuum.

Our paper consists of the following sections. Several observations of the fine structure in the solar radio emission of type a quasi-periodic millisecond modulation, are given in Section 2. The common characteristics of these fine structures have been also summarized in this section. A nonlinear propagation equation of electromagnetic wave and the necessary condition of a longitudinal modulational instability of this equation are presented in Section 3. The propagation effects of electromagnetic pulse from corona to the observer on the Earth has been briefly studied in Section 4. The possibilities of an occurrence of fine structures as a quasi-periodic millisecond modulation in the solar X ray, optical, radio emission and some concluding remarks have been also given in the last section.

## 2. Observation

Several quasi-periodic millisecond fine structures in the metric wave band occurring during the evolution of solar radio type IV bursts were observed by a few solar radio telescopes with high time resolution (< 15 ms). An event of such fine structure around the 290 MHz occurring during a solar radio burst, was observed by Yunnan Observatory on Aug. 23, 1990 (Xia *et al.*, 1993), as shown in Figure 1. The corresponding radio burst was also observed in the different frequency band by Weissenau spectrometer. The duration of these fine structures are about 500 ms, with an oscillation period from 43 ms to 108 ms. The envelope of these fine structures are not a spiky or a zebra fine structure. Meanwhile, another event of milliseconds quasi-

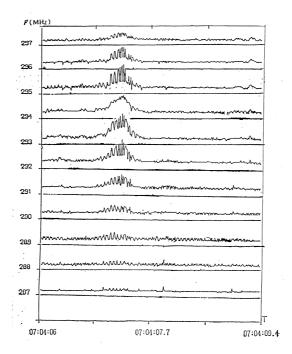


Figure 1. The ms envelope soliton fine structure in the event on 23 Aug., 1990, observed from the metric spectrometer of Yunnan Observatory (courtesy from Xia et al., 1993).

periodic fine structure around 1415 MHz in solar radio emission was observed by Yunnan Observatory on Jan. 6, 2000. (Xia, 2000).

Recently, two events of quasi-periodic, milliseconds fine structure with right circular polarization at 237 MHz were found by Trieste Radio telescope and IZMIRAN dynamic spectrometer, as shown in Figure 2 and Figure 3. Such modulation happens against in the duration of solar radio burst event on June 5, 1990, in the decimetric and metric continuum emission ('type IV burst'). Some zebra patterns and fiber bursts in the same event have been observed by several solar radio instruments simultaneously. These fine structures have been analyzed by Chernov *et al.* (1994), in detail. The millisecond quasi-periodic fine structures in the same event should provide more reliable and precise pattern than Figure 1. It will be a great help to determine the mechanism of such fine structures in the solar radio emission.

As we can see in Figure 2 and Figure 3, some distinguished parts (by straight lines) present a modulation of the continuum emission with a very fast rate (about 10 pulsations per second) and with a maximum modulation depth ( $\simeq 30\%$ ) in the center of each fine structure. These fine structures have about 0.7-1.5 s duration. At Figure 2, a such modulation takes place at the type IV continuum emission background about 200 (sfu), and at Figure 3 at the level about 40 (sfu).

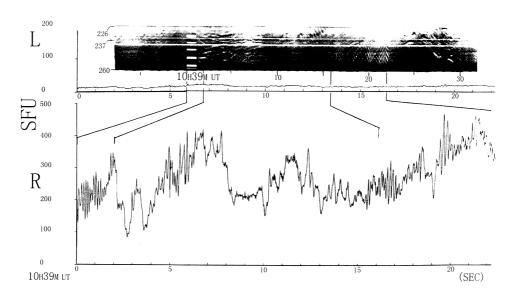
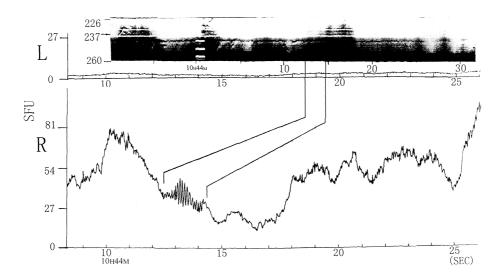


Figure 2. A solar radio burst event of type IV continuum spectrum on 5 June, 1990 was observed by IZMIRAN radio spectrometer from 226 MHz to 260 MHz (top), and Trieste radio telescope at 237 MHz (L: left circular polarization; R: right circular polarization). The envelope soliton fine structure displays in R, at 10:39:01 UT.



*Figure 3.* Another envelope soliton fine structure displays at 10:44:13 UT in R of the same event on 5 June, 1990, as in Figure 2.

At the dynamical spectra these parts correspond to very fast zebra-pattern with the frequency drift of zebra lines about  $-40 \text{ MHz s}^{-1}$ .

Another quasi-periodic, (-60 ms) milliseconds fine structure event with right circular polarization at 1420 MHz was also found by Ondřejov radiospectrograph. This fine structure occurs in the duration of solar radio burst on March 18, 1999. The duration of this event is about 600 ms and its modulation envelope is also like to a soliton pattern (Karlický, 1999).

These millisecond fine structures of solar radio emission have several common characteristics as follows:

- (a) They were only observed in the frequency region of 200 1500 MHz up to now, corresponding to the metric wave and longer decimetric wave.
- (b) Most durations of these fine structures are less than one second, it is a milliseconds fine structure in solar radio emission.
- (c) They have a quasi-periodic modulation, the modulation period is  $\sim 100$  ms, and the modulational fine structure with a larger amplitude has a shorter modulation period.
  - (d) The envelope of these modulational fine structures have a soliton pattern.
- (e) These fine structures with larger amplitude often possess a narrower width, as shown in Figure 2 and 3.
- (f) The strong circular polarization of these fine structures were observed in the three events, all of them are in right sign as for the type IV continuum radio emission.
- (g) These fine structures often display in the duration of solar radio type IV burst event.

Among them, the characteristic (c), (d) and (e) have not been found in other solar radio milliseconds fine structures, but these three characteristics are very similar to the envelope soliton structures, which were detected by satellites in different space region (Gurnett *et al.*, 1981; Eriksson *et al.*, 1994; Wang *et al.*, 2002). Therefore, we call such particular milliseconds fine structures as an 'envelope soliton-like', like in the space plasma research.

However, it should be noted that there are three differences for the millisecond modulation fine structure in the observation between satellites and solar radio instruments on the Earth:

- (1) The electric field (positive and negative) can be directly measured in satellites, it is related to the amplitude of modulational instability; but only solar radio emission flux and polarization may be measured on the Earth.
- (2) The depressive plasma density soliton (caviton) always accompanies with the envelope soliton of electric field amplitude, which was confirmed in the observation from satellites. However, it is impossible to detect simultaneously the coronal plasma density variation with envelope soliton-like of flux in the solar radio emission.
- (3) It is required to consider the propagation effects on the envelope soliton-like from corona to the observer on the Earth.

Therefore, it should be more difficult to determine envelope soliton-like fine structure in the solar radio observation.

### 3. Modulational Instability of Electromagnetic Waves in the Solar Corona

In order to explain these envelope soliton-like fine structures, we consider that the electromagnetic waves in the solar corona are modulated by a low frequency wave, which will give rise a modulational instability. For sake the simplicity, it is assumed that the wave propagates along the magnetic field  $B_0$  in the z direction, and coronal plasma is homogeneous. The electric field of an electromagnetic wave  $(\omega, k)$  packet can be written as

$$\mathbf{E} = \Psi \exp[i(k_z z - \omega t)] + c.c. \tag{1}$$

where  $\Psi$  is a slow variation function of  $\mathbf{r}$  and t. In the approximation of geometry optics, the nonlinear wave propagation equation in the plasma can be expressed as (Karpman, 1975)

$$i\left(\frac{\partial}{\partial t} + V_g \frac{\partial}{\partial z}\right) \Psi + \frac{1}{2} V_g' \frac{\partial^2}{\partial z^2} \Psi + \frac{1}{2} T \frac{\partial^2}{\partial x^2} \Psi - \Delta \omega \Psi = 0$$
 (2)

where  $V_g=\frac{\partial \omega}{\partial k}$  is the wave group velocity,  $V_g'=\frac{\partial V_g}{\partial k}$  is the dispersion of wave group velocity,  $T=\left(\frac{\partial^2\omega}{\partial k_\perp^2}\right)_{k_\perp}=0$  is the perpendicular dispersion,  $\Delta\omega$  is the nonlinear frequency shift.

The consideration of the observation of events with the envelope soliton-like fine structure displays only the right circular polarization (the same one with the continuum emission, as shown in Figure 2 and Figure 3), and the solar radio type IV bursts are likely to be ordinary (O) mode emission (Benz, 1993; Chernov *et al.*, 1994). We consider the dispersion relation of an O-mode electromagnetic wave in the cold plasma approximation, which can be written as

$$N^2 = \frac{c^2 k^2}{\omega^2} = 1 - \frac{R}{u(u+1)} \tag{3}$$

where N is the refraction index of coronal plasma,  $R = \frac{\omega_{pe}^2}{\omega_{ce}^2}$ ,  $u = \frac{\omega}{\omega_{ce}}$  are dimensionless parameters.  $\omega_{pe}$  is the electron plasma frequency, and  $\omega_{ce}$  is the electron cyclotron frequency. Substituting the dispersion relation (3) into the expressions of  $V_g$ ,  $V_g'$ , T, and  $\Delta \omega$ , we obtain:

$$V_g = \frac{2cN}{2 + \frac{R}{u(u+1)^2}} \tag{4}$$

$$V_g' = \left[1 - \left(\frac{V_g}{c}\right)^2 \left(1 + \frac{R}{(u+1)^3}\right)\right] \left(\frac{V_g}{k}\right) \tag{5}$$

$$T = \left[1 - \frac{R}{2(u^2 - R)(u + 1)}\right] \left(\frac{V_g}{k}\right). \tag{6}$$

It is further assumed that a perturbation low frequency wave is  $(\Omega, \mathbf{K})$ , which is induced by the high frequency wave (Swanson, 1989). In the case of one MHD fluid model, the nonlinear frequency shift  $\Delta \omega$ , in the coordinate of  $S = K_x x + K_z z - \Omega t$  can be expressed as (Karpman and Washimi, 1977):

$$\Delta\omega = \frac{G(\Omega, K)}{D(\Omega, K)} \Phi^2 \tag{7}$$

$$G(\Omega, K) = -\left(\frac{kV_g V_A^2}{8}\right) [\Omega^2 - V_A^2 K^2) (d_1 - \mu d_2)^2 + K_x^2 K_z^2 C_x^2 d_2^2 (1 - \mu)^2 - \Omega^2 K_x^2 d_3^2]$$
(8)

$$D(\Omega, K) = (V_A^2 K^2 + C_s^2 K_x^2 - \Omega^2)(C_s^2 K_z^2 - \Omega^2) - C_s^2 K_x^2 K_z^2$$

where

$$d_{1} = \frac{N^{2} - 1}{N^{2}} = \frac{R}{R - u(u + 1)}$$

$$d_{2} = \frac{\partial [\omega^{2}(N^{2} - 1)]}{N^{2}\omega\partial\omega} = \frac{R}{(u + 1)[u(u + 1) - R]}$$

$$d_{3} = \frac{\partial [\omega(N^{2} - 1)]}{\omega\partial\omega} = d_{2} - d_{1} = \frac{Ru}{(u + 1)[u(u + 1) - R]}$$
(9)

where  $\mu = \frac{\Omega k}{\omega K_z}$ ,  $\Phi = \frac{N\Psi}{B_0}$ . It should be noted that the nonlinear frequency shift is proportional to the square of propagation wave amplitude. Therefore, the Equation (2) is a nonlinear parabola equation, (or it is called nonlinear Schrödinger equation).

A perturbation method has been adopted in order to analyze the unstable branches of Equation (2), called modulational instability, (Karpman, 1975), we obtain

$$(\Omega - K_z V_g)^2 - \frac{1}{4} (V_g' K_z^2 + T K_x^2)^2 = \frac{G(\Omega, K)}{D(\Omega, K)} (V_g' K_z^2 + T K_x^2) \Phi_0^2$$
 (10)

where  $\Phi_0$  is the unperturbed amplitude.

In the case of longitudinal modulation, it corresponds to  $K_x = 0$ . The low frequency wave, same as the electromagnetic wave, also propagates along the z direction. We can obtain the necessary condition of instability from Equation (10)

$$(\Omega - K_z V_g)^2 = \frac{1}{4} (V_g' K_z^2)^2 + \left(\frac{k V_g V_A^2}{8}\right) \frac{(d_1 - \mu d_2)^2 V_g' K_z^4}{\Omega^2 - C_z^2 K_z^2} \Phi_0^2.$$
 (11)

It is due to  $V_g'>0$  in the case of u>1; thus the instability only occurs in the situation of  $\Omega < C_s K_z$ . It means that the Alfvén velocity  $V_A = \frac{\Omega}{K_z}$  should be less than the sound velocity  $C_s$ , if the perturbation low frequency wave is an Alfvén wave. It corresponds to in the coronal region of the low magnetic field and the high temperature, as well as the high density plasma; or a longitudinal modulational instability only occurs in the high  $\beta$  coronal region, which can exist in the outer of corona. (>  $5 \times 10^6$  km, see e.g. Figure 7 in Rosenberg, 1973), since at the low and middle corona above the active regions the  $\beta$  value of plasma should be usually less than 1. In the other words, the envelope soliton-like fine structure should only occur in the solar radio burst of metric band waves (including longer decimetric waves and shorter decametric waves).

It is of interest to note that the necessary condition of electromagnetic waves for a longitudinal modulational instability is different with the necessary condition of whistler wave and ion cyclotron wave. The latter is due to the negative dispersion of group velocity (Karpman and Washimi, 1977); but the former is due to negative nonlinear frequency shift.

In this case of longitudinal modulational instability, the Equation (2) has a stationary solution. It is given by (Hasegawa, 1975)

$$\Psi = \Psi_m \operatorname{sech}(\Psi_m \kappa^{\frac{1}{2}} \xi) \exp \left[ i \left( \frac{1}{2} \Psi_m^2 \kappa \tau \right) \right]$$
(12)

where  $\tau = V_g't$ ,  $\xi = x - V_gt$ ,  $\kappa = -\frac{G(\Omega,K)N^2}{D(\Omega,K)V_g'B_0^2}$  and  $\Psi_m$  is the maximum value of the electric field amplitude. It will produce a fine structure of envelope soliton-like pattern in the emission flux  $I = |\Psi|^2$ . For example, we may choose the plasma parameters in the high corona region as follows: the magnetic field  $B_0 = 1$  Gauss, the density of plasma  $n_0 = 2.4 \times 10^7$  cm<sup>-3</sup>, the temperature T = 2.3 keV. The  $V_g$ ,  $V_g$ ,  $V_A$ ,  $C_s$ ,  $d_1$ ,  $d_2$ ,  $d_3$  etc. can be calculated from these parameters and observational data. We can obtain the estimated values:  $V_g = 5.7 \times 10^9$  cm s<sup>-1</sup>,  $V_g' \simeq \frac{V_g}{k} = 3.4 \times 10^{12}$  cm<sup>2</sup> s<sup>-1</sup>,  $k = 1.7 \times 10^{-2}$  cm<sup>-1</sup>,  $C_s = 4.7 \times 10^7$  cm s<sup>-1</sup>, and  $V_A = 4.4 \times 10^7$  cm s<sup>-1</sup>. The sound velocity  $C_s$  can be larger than the Alfén velocity  $V_A$ . It is consistent with the observational data rather well. Meanwhile, from Equation (12), the amplitude of envelope soliton are inversely proportional to the width of envelope soliton and are also inversely proportional to the modulation period, these characteristics are consistent qualitively with the observational character (e) and (c) in Section 2. Hence, we think that the observed envelope soliton-like fine structures in solar radio emission, may be caused by the longitudinal modulational instability in the high coronal plasma region.

# 4. Envelope Soliton-Like Propagating in the Solar-Terrestrial Space

As mentioned in Section 2, the propagation effects of an envelope soliton-like fine structure from corona to the observer on the Earth should be further considered. It is convenient to decribe an envelope soliton-like as an electromagnetic pulse. The diffraction divergence and dispersion broadening effects of an electromagnetic pulse propagation in a homogeneous plasma have been discussed by Liu and Tripathi (1994). It was found that the electromagnetic pulse will diverge in the perpendicular direction and broaden along the propagation. The amplitude of the electromagnetic pulse will decrease during the propagation, simultaneously.

It is found that an envelope soliton-like will also broaden during it propagates in an inhomogeneous solar-terrestrial plasma. It is assumed that the distribution of the plasma density n(z) in the solar-terrestrial space can be approximately written as

$$n(z) = n_0 \left( 1 + \frac{n_1(z)}{n_0} \right) = n_0 (1 + \alpha z). \tag{13}$$

The  $\alpha$  is a negative value, if the plasma density linearly decreases with the distance from corona. The nonlinear Schrödinger equation of a slow variation amplitude  $\Psi$  of the wave in a weakly inhomogeneous medium can be described as

$$i\frac{\partial\Psi}{\partial t} + \frac{\partial^2\Psi}{\partial z^2} + 2(|\Psi|^2 - \alpha z)\Psi = 0. \tag{14}$$

This equation was shown to be integrable. (Chen and Liu, 1976; 1978), it has a solution of

$$\Psi(z,t) = 2\eta \operatorname{sech} 2\eta (z + 4\xi t + 2\alpha t^{2} - z_{0})$$

$$\exp\{-i[2(\xi + \alpha t)z + \frac{4}{3}\alpha^{2}t^{3} - 4\alpha\xi t^{2} + 4(\xi^{2} - \eta^{2})t + \Psi_{0}]\}$$
(15)

where  $\eta$  and  $\xi$  are the amplitude and initial speed of soliton, respectively. We can find that the width of the envelope soliton-like is inversely proportional to the value  $(z-z_0+4\xi t+2\alpha t^2)$ . Therefore, in the case of the plasma density in the solar-terrestrial space linearly decreases with the distance from corona, the envelope soliton-like fine structure should be stable during it propagates from corona to the observer on the Earth, but the soliton will broaden. However, it is generally impossible to describe the density variation in the solar-terrestial space as a linear decrease with the distance from corona, in fact, the density behaves like  $r^{-2}$  decreasing in the interplanetary space. This problem is still open. The propagation effects of an envelope soliton-like from corona to the observer on the Earth should be investigated in more real model.

### 5. Discussion

- (a) It has seen from the discussion of Section 3, the necessary condition of modulational instability depends on the coronal plasma parameters. Dulk and Mclean (1978) provided an estimate value of magnetic field from solar radio bursts. As a general rule, the Alfvén velocity is often larger than sound velocity  $V_A > C_s$  in the inner coronal region. Therefore, we expect from our theoretical model that the optical, X ray and high frequency electromagnetic waves, which produced in the inner coronal region, will not occur such envelope soliton-like fine structure. On the other hand, the frequencies of high frequency electromagnetic waves, optical and X ray are much larger than the electron cyclotron frequency in corona, so that  $u(u+1) \gg R$ . In this case, the dispersion of group velocity  $V'_g \to 0$  and the value of  $d_1 \sim d_2 \to 0$ . It can be readily to see from (11) that longitudinal modulational instabilities is impossible to occur in the inner coronal region with strong magnetic field.
- (b) Chernov *et al.* (1994) have found that different types of fine structure (fast pulsations, zebra pattern) come from different location within a great magnetic structure. This event was not connected with a big flare, and the radio source was located far from the active region, but just above a dark filament. We find from Figure 2 and Figure 3 that these envelope soliton-like fine structures do not display at the maximum flux of type IV burst. It seems that the modulational instability also only occurs in the small region (sub-source) of a global type IV radio source, as other fine structures. However, it is required to further confirm from solar radioheliograph and more observational events.
- (c) It may be adopted two-fluid MHD model to deal with the low frequency waves, which will induce an ion-acoustic wave perturbation along the magnetic field. We find that the nonlinear frequency shift can be written as

$$\Delta\omega = \frac{kV_g V_A^2 K_z^2}{8(\Omega^2 - U_s^2 K_z^2)} (d_1 - \mu d_2)^2 \Phi^2$$
 (16)

where  $U_s = \left(\frac{T_e}{m_i}\right)^{\frac{1}{2}}$  is ion-acoustic velocity. In consideration of  $V_g' > 0$  for the electromagnetic waves, it is required the  $\kappa > 0$  in (12), thus the longitudinal modulation is stable for the ion-acoustic wave perturbation. Shukla and Stenflo (1984) have considered the combined effects of the relativistic mass and ponderomotive force nonlinearities in the nonlinear propagation of an electromagnetic wave, it can induce a longitudinal modulational instability, but the relativistic mass effect is unimportant in the solar radio emission.

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